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Mechanics-based Approach for Estimating Force Distribution to Lateral Load Resisting Elements in Timber Buildings

KEYWORDS: Lateral Load Resisting Elements (LLRE); Timber Buildings; Diaphragms

OVERVIEW OF PROJECT

In timber structures that contain a number of different lateral load resisting elements (LLRE's), the distribution of lateral loads arising from wind and earthquake to these elements depends on the stiffness characteristics of the horizontal diaphragm that are attached to these LLRE's relative to those of the LLRE's. In structural design, for simplicity horizontal diaphragms are often treated as either rigid or flexible. Whilst the assumption of rigid or flexible diaphragms is convenient from a design perspective, the reality is that the diaphragms of timber structures are often semi-rigid, which significantly complicates the calculation of forces resisted by LLRE's. The objective of this project is to develop a mechanics-based approach for estimating the distribution of design lateral force to LLRE's taking into consideration the stiffness of diaphragm.

A multiple spring model (MSM), as shown in Fig. 1, was proposed to simulate a single-storey building consisting of several LLRE's. The reactions and deformations of the LLRE's were derived by solving the system of equations using the Gaussian Elimination Technique of matrix structural analysis. Three timber buildings with different layouts were modelled using the finite element software, ABAQUS, to verify the MSM by comparing the force distributions in the LLRE's.

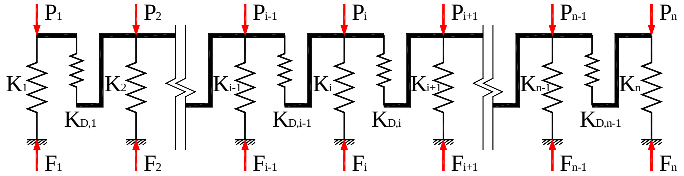


Fig. 1. Multiple spring model for a single-storey building

KEY RESULTS

In the MSM, each LLRE is represented by a spring with K_i connected to the ground, and the diaphragm between adjacent LLREs is simulated by a spring with K_{Di} connected to the springs of the two adjacent LLRE's via a rigid beam with only one degree of freedom in the direction of the applied load (P_i). The reactions (F_i), which essentially represent the forces absorbed by the LLRE's, and deformations (u_i) of the LLRE springs in this model can be obtained by solving the system of equations with 2n variables, as shown in Eq. (1).

$$\left\{F\right\}_{2n} = \left[D\right]_{2n\times 2n}\left\{U\right\}_{2n} \qquad (1)$$

where *n* is the number of LLRE's, and

$$\left\{F\right\}_{2n} = \left\{F_1 \quad -P_1 \quad \cdots \quad F_i \quad -P_i \quad \cdots \quad F_n \quad -P_n\right\}^T_{2n} \qquad (2)$$

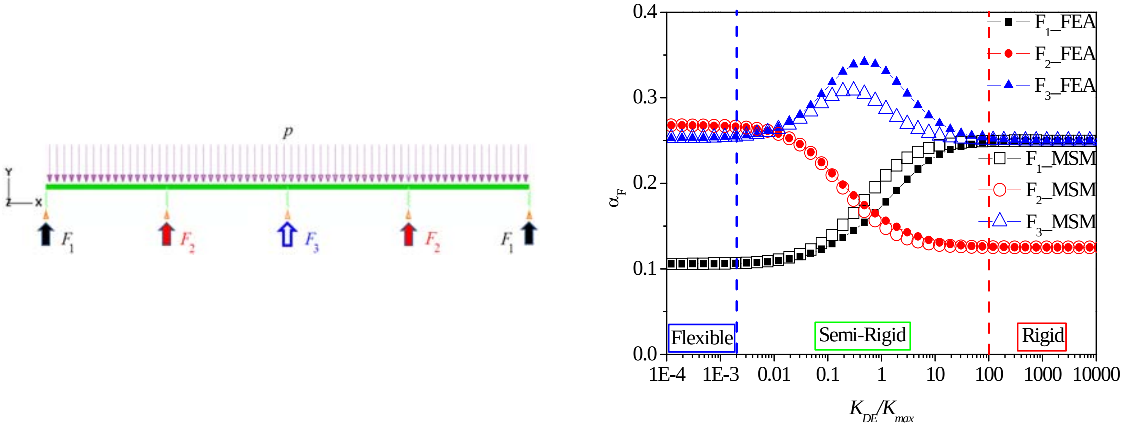
Researchers: Zhiyong Chen[1], Ying-Hei Chui[1], Chun Ni[2], Ghasan Doudak[3], Mohammad Mohammad[2]
Affiliation: [1]University of New Brunswick, [2]FPInnovations, [3]University of Ottawa
Contact email address: zhiyong.chen@unb.ca

$$[D]_{2n\times 2n}=\begin{bmatrix}\ddots & 0 & \vdots & 0 & 0 & 0\\ 0 & K_i & -K_i & 0 & 0 & 0\\ \cdots & -K_i & K_i+K_{D,(i-1)}+K_{D,i} & 0 & -K_{D,i} & 0\\ 0 & 0 & 0 & K_{(i+1)} & -K_{(i+1)} & 0\\ 0 & 0 & -K_{D,i} & -K_{(i+1)} & K_{(i+1)}+K_{D,i}+K_{D,(i+1)} & \cdots\\ 0 & 0 & 0 & 0 & \vdots & \ddots\end{bmatrix}_{2n\times 2n} \quad (3)$$

$$\{U\}_{2n}=\{0\quad u_1\quad \cdots\quad 0\quad u_i\quad \cdots\quad 0\quad u_n\}^T_{2n} \quad (4)$$

The solution procedure can be easily implemented in a simple computer program, or using an Excel spreadsheet. A copy of the program can be obtained from the authors. For light wood frame buildings input stiffness properties (K_i and K_{Di}) of the diaphragm segments and LLRE's can be back-calculated from the deflection equations provided in design standards, such as CSA O86, for horizontal diaphragm and shear wall. Due to the nonlinear nature of load-deformation responses of shear walls and diaphragms, the stiffnesses were taken as the secant slope between the zero-load point and the design-resistance point of the curves determined by the CSA O86 equations. The total vertical elongation of the shear wall anchorage system, d_a, and the chord-splice slip, Δ_C, of the diaphragm are assumed to be proportional to the applied load.

The model was verified with results from finite element analyses (FEA). A single-storey building containing five shear walls with three stiffness combination cases was considered. The FEA model is illustrated in Figure 2(a). Results are shown in Figure 2(b), where α_F is the ratio of force in LLRE over the total applied force and $K_{DE}/K_{\max}$ is the ratio of equivalent stiffness of the diaphragm over maximum LLRE stiffness. An interesting observation in the results shown in Fig. 2b is that the force ratio of some of the internal springs was higher than the values at the two extreme regions, i.e. rigid and flexible. This phenomenon is thought to be induced by deformation compatibility that occurs in continuous beams. This finding raises the concern that the envelope force approach commonly used by designers (i.e. taking the larger of the shear wall force based on either flexible or rigid diaphragm) may lead to an underestimation of design forces since diaphragms are generally semi-rigid.



(a) FEA model of a single-storey building

(b) Force ratio vs. stiffness ratio

Fig. 2. FEA model and lateral force distribution