WCTE 2014 Quebec City Canada August 10-14
World Conference on Timber Engineering

LOAD DISTRIBUTION IN LATERAL LOAD RESISTING ELEMENTS OF TIMBER STRUCTURES

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ABSTRACT: Stiffness properties of diaphragm and lateral load resisting elements (LLREs) influence the load distribution between LLREs under lateral load induced by earthquake or wind. Where a more sophisticated method of calculating the load distribution in a lateral load resisting system is used, often it is based on the concept of beam on elastic foundation. This approach could be tedious to apply in design when there are several LLREs. Two multiple spring models (MSMs) for estimating the load distribution in LLREs of timber structures are proposed. One MSM without rotational degree of freedom is only consisted of translational springs by which the shear wall and diaphragm segments are simulated; and the other MSM with rotational degree of freedom, in which the shear wall segments are modeled by translational springs and the diaphragm segments are simulated by translational and rotational springs, is able to take into account the torsional effect on a building's lateral load response. The developed models were validated with results from a more sophisticated analysis using a finite element program. The lateral load distribution between LLREs with various stiffness ratios of diaphragm to LLREs and considering the torsional effects was also investigated. The results show that, contrary to common belief, the forces transferred by a semi-rigid diaphragm to the supporting LLREs may be higher than those predicted by flexible and rigid diaphragm assumptions. Hence using the envelope force approach proposed by some may lead to underestimation of the design forces in the shear walls.

KEYWORDS: Timber structures, Structural elements, Diaphragms, Lateral load resisting element, Load distribution.

1 INTRODUCTION

The flexibility of diaphragms, whether in or out of plane has significant effects on the structural behaviour of buildings. For instance, the rotation of shear walls is influenced by the out-of-plane diaphragm flexibility [1], whereas the in-plane diaphragm flexibility affects the load distribution between shear walls of the light wood frame buildings [2]. In timber structures that contain a number of different lateral load resisting elements (LLREs), the distribution of the lateral loads arising from wind and earthquake is dependent on the stiffness characteristics of the horizontal diaphragm relative to those of the LLREs supporting the diaphragm.

Figure 1 shows a single-storey timber building under a uniform load, *p*, where the diaphragm acts as a load distributor to the LLREs that run parallel to the direction of the applied lateral load. In design, if the diaphragm is idealized as 'rigid', all LLRE's are assumed to deform by the same amount, and therefore the lateral load is distributed to the LLREs in proportion to their stiffness. As a result, a stiffer LLRE would attract a higher proportion of the applied lateral load. If the diaphragm is assumed 'flexible', the LLREs deform by different amounts and the lateral load resisted by each LLRE will be assigned based on the respective tributary area. While the assumptions of rigid and flexible diaphragms are convenient from a design perspective, reality is that wood-based diaphragms are often semi-rigid, which significantly complicates the calculation of forces resisted by LLREs [3, 4].

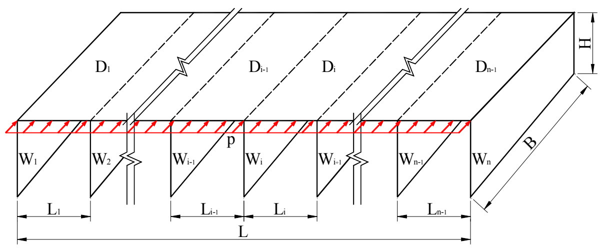

Figure 1: *Single-storey timber building*

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To assist with the design process, guidance on classifying diaphragms into various stiffness categories has been enacted by some design standards, including Eurocode 8 [5], ASCE 07-10 [6], IBC [7] and ASCE 41-06 [8]. For example, ASCE 41-06 states that a diaphragm shall be classified as flexible when $\Delta_{D,\max}/\Delta_{L,\text{ave}}$ is larger than 2.0, where $\Delta_{D,\max}$ is the maximum horizontal deformation of the diaphragm and $\Delta_{L,\text{ave}}$ is the average inter-storey drift of the LLREs of the storey immediately below the diaphragm. Rigid diaphragm is defined where $\Delta_{D,\max}/\Delta_{L,\text{ave}}$ is smaller than 0.5. ASCE 41-06 notes that the diaphragm can be regarded as semi-rigid if $\Delta_{D,\max}/\Delta_{L,\text{ave}}$ is between 0.5 and 2.0, however, no further guidance is provided on how the forces are distributed in this case. Other national standards, such as the Canadian timber design standard CSA O86-09 [9], are silent on diaphragm flexibility classification, and assigning forces to the shear walls is left to the discretion of the designers. A document published by the Association of Professional Engineers and Geoscientists of British Columbia (APEGBC) [10] recommends that an envelope approach should be used if the force in any wood shear wall differs by more than 15% due to the change in the flexible and rigid diaphragm assumptions, which was adopted from a seismic design manual [11]. This approach is simple and is thought to yield conservative results.

To date, limited research [4, 12-15] has been undertaken to systematically evaluate the diaphragm flexibility and its influence on the load distribution to LLREs. The authors of this paper have previously proposed a multiple-spring model (MSM) for calculating the load distribution to the LLREs with different stiffness [2]; however, this approach was limited to building configurations with symmetrical arrangement of LLREs about the direction of lateral load, and it did not take into account the torsional effect that happens in asymmetrical cases. In this study, a modified MSM is proposed for estimating the load distribution between LLREs with considerations given to the torsional effect.

2 MULTIPLE SPRING MODEL (MSM)

The beam-on-elastic-foundation concept is an established approach to estimate the distribution of lateral loads to individual LLREs. Under this approach, the diaphragm is modelled as a deep beam acting as a load transfer mechanism [4, 12, 14, 16]. The stiffness of the diaphragm is represented by the flexural and shear rigidity of the elastic deep beam, while the LLREs are modelled using a series of linear springs. Using this deep beam-on-springs model (DBSM), as shown in Figure 2, the distribution of the reaction forces in the spring supports represent the forces resisted by the LLREs.

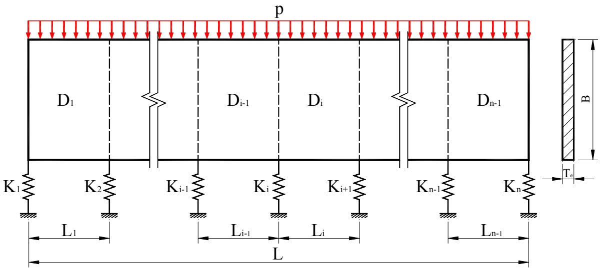


Figure 2: *Deep beam-on-spring model*

For a simple building with only a few LLREs, solutions of the DBSM for design purposes can be obtained with relative ease using basic mechanics. However, as the number of diaphragm elements and LLREs increases, it becomes more tedious, and more sophisticated analysis procedures, such as finite element analysis (FEA), may be required.

To provide an alternative method to establish the diaphragm-LLRE models and estimate the load distribution within a matrix structural analysis framework, a modified multiple spring model (MSM) is proposed to take into account the torsional effect based on the MSM without rotational degree of freedom (DOF) [2]. These two models are presented and compared in this study.

2.1 MSM WITHOUT ROTATIONAL DOF

In the MSM without rotational DOF [2], as illustrated in Figure 3 for a single-storey building, each LLRE is represented by a translational spring with stiffness K_i connected to the ground, and every diaphragm segment between adjacent LLREs is represented by a translational spring with stiffness K_{Di} connected to the springs of the two adjacent LLREs via a rigid beam with only one translational DOF in the direction of the applied load.

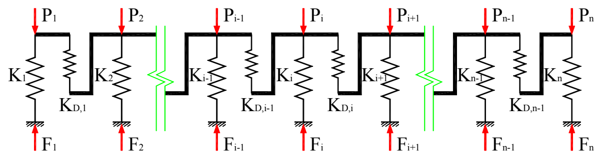


Figure 3: *Multiple-spring model without rotational DOF*

The reactions, F_i, and deformations, μ_i, of the LLRE springs in this MSM can be obtained by solving the system of equations with 2n variables, as shown in Equation (1).

$$\{F\}_{2n} = [D]_{2n\times 2n}\{U\}_{2n} \tag{1}$$

where,

$$\{F\}_{2n} = \{\cdots \quad F_i \quad -P_i \quad \cdots\}_{2n}^T \tag{2}$$

$$\{U\}_{2n} = \{\cdots \quad 0 \quad u_i \quad \cdots\}_{2n}^T \tag{3}$$

$$[D]_{2n\times 2n} = \begin{bmatrix} \ddots & 0 & \cdots & 0 & 0 & 0 \\ 0 & K_i & -K_i & 0 & 0 & 0 \\ \vdots & -K_i & K_i + K_{D,(i-1)} + K_{D,i} & 0 & -K_{D,i} & 0 \\ 0 & 0 & 0 & K_{(i+1)} & -K_{(i+1)} & 0 \\ 0 & 0 & -K_{D,i} & -K_{(i+1)} & K_{(i+1)} + K_{D,i} + K_{D,(i+1)} & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \ddots \end{bmatrix}_{2n\times 2n} \tag{4}$$

As it can be seen from Figure 3 and Equations (1) to (4), only the transitional springs are used for the MSM, and thus the rotational effect of diaphragm is not included. This MSM is therefore suitable for symmetrical cases or asymmetrical LLRE cases of which the torsional effect on the load distribution between LLREs is not significant.

2.2 MSM WITH ROTATIONAL DOF

For some timber building cases, however, the LLREs would not be designed symmetrically, in which the torsional effect would be induced and thus affect the load distribution between the LLREs. In order to take the torsional effect into account, a modified MSM with rotational DOF, Figure 4, was established by adding rotational springs, $K_{\theta,i}$, to the two ends of diaphragm segments (Figure 3).

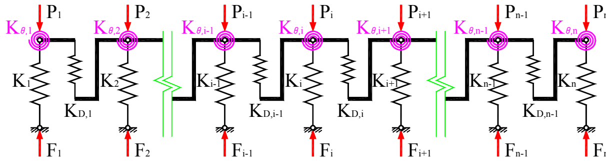


Figure 4: *Multiple-spring model with rotational DOF*

Similar to the MSM without rotational DOF, the uniform load, p, shown in Figure 1 is converted into a concentrated load, P_i, acting at the top of each LLRE, and calculated using the product of the uniform load, p, and the tributary area, A_i, of the corresponding LLRE, as shown in Equation (5).

$$P_i = A_i p \tag{5}$$

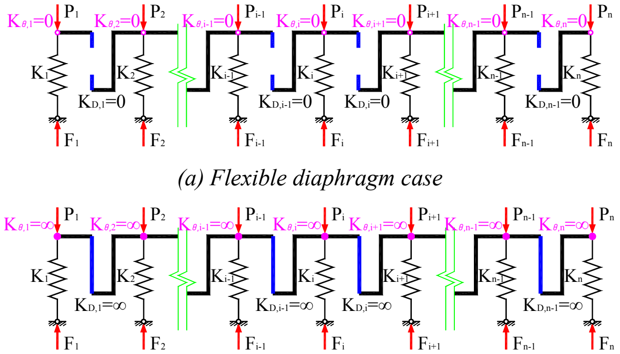
The deformation compatibility of the diaphragm is modelled via the diaphragm transitional and rotational stiffness, $K_{D,i}$ and $K_{\theta,i}$, which can be taken as the reciprocal of their deformations under the load similar to the LLRE stiffness K_i. If the diaphragm is assumed flexible, the stiffness of the diaphragm transitional and rotational springs, $K_{D,i}$ and $K_{\theta,i}$, are taken as zero, therefore, there will be no connection between the two adjacent LLRE springs (Figure 5a). The force in the LLRE spring, F_i, is therefore proportional to the tributary area of the corresponding LLRE, Equation (6).

$$F_i \propto \alpha_{A,i} = A_i \Big/ \sum_{i=1}^{n} A_i \tag{6}$$

where $\alpha_{A,i}$ is the tributary area ratio. When the diaphragm is assumed rigid, then $K_{D,i}$ and $K_{\theta,i}$ are taken as infinity (Figure 5b), and each LLRE resists the load, F_i, in proportion to the relative stiffness of the LLREs, Equation (7).

$$F_i \propto \alpha_{K,i} = K_i \Big/ \sum_{i=1}^{n} K_i \tag{7}$$

where $\alpha_{K,i}$ is the stiffness ratio.



(a) Flexible diaphragm case

(b) Rigid diaphragm case

Figure 5: *Two extreme cases of multiple spring model*

The reactions, F_i, and translations, μ_i, of the LLRE springs, and the rotations, θ_i, of diaphragm segment springs in this MSM with rotational DOF can be derived by solving the system of equations with 3n variables, as shown in Equation (8).

$$\{F\}_{3n} = [D]_{3n\times 3n}\{U\}_{3n} \tag{8}$$

where,

$$\{F\}_{3n} = \{\cdots \quad F_i \quad -P_i \quad 0 \quad \cdots\}^T_{3n} \tag{9}$$

$$\{U\}_{3n} = \{\cdots \quad 0 \quad u_i \quad \theta_i \quad \cdots\}^T_{3n} \tag{10}$$

Equation (8) can be solved by applying the Gaussian Elimination Method. The solution routine can be implemented in a simple computer program. The benefits of the proposed MSM over the DBSM are two-fold. First, stiffness property for input into the translational spring of each diaphragm segment and LLRE is easily obtainable since design standards typically provide equations for calculating deformation of diaphragm and shear wall. Second, the solution procedure can be easily implemented into a simple computer program, or by using an Excel spreadsheet.

$$[D]_{3n\times 3n} = \begin{bmatrix} \ddots & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ 0 & K_i & -K_i & 0 & 0 & 0 & 0 & 0 \\ \vdots & -K_i & K_i + K_{D,(i-1)} + K_{D,i} & \left(-L_{(i-1)}K_{D,(i-1)} + L_iK_{D,i}\right)/2 & 0 & -K_{D,i} & \left(L_iK_{D,i}\right)/2 & 0 \\ \vdots & 0 & \left(-L_{(i-1)}K_{D,(i-1)} + L_iK_{D,i}\right)/2 & K_{\theta,(i-1)} + K_{\theta,i} & 0 & \left(-L_iK_{D,i}\right)/2 & K_{\theta,i}/2 & 0 \\ 0 & 0 & 0 & 0 & K_{i+1} & -K_{i+1} & 0 & 0 \\ 0 & 0 & -K_{D,i} & \left(-L_iK_{D,i}\right)/2 & -K_{i+1} & K_{i+1} + K_{D,i} + K_{D,(i+1)} & \left(-L_iK_{D,i} + L_{(i+1)}K_{D,(i+1)}\right)/2 & \cdots \\ 0 & 0 & \left(L_iK_{D,i}\right)/2 & K_{\theta,i}/2 & 0 & \left(-L_iK_{D,i} + L_{(i+1)}K_{D,(i+1)}\right)/2 & K_{\theta,i} + K_{\theta,(i+1)} & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & \ddots \end{bmatrix}_{3n\times 3n} \tag{11}$$

$$K_{\theta,i} = \left(L_i^2 K_{D,i}\right)/3 \tag{12}$$

3 LOAD DISTRIBUTION IN LLREs

3.1 ANALYSIS CASES

The application of the MSM without rotational DOF was performed on a two-storey NEESWood benchmark building, where its seismic performance was investigated at full scale by conducting a series of shake table tests [2]. Similar application of the modified MSM with rotational DOF proposed in this study can be conducted as well. In this study, however, five cases (Figures 6 and 7) of single-storey building with and without consideration of the torsional effect were analysed under the lateral load, to verify the proposed modified MSM and investigate the load distribution between LLRE's.

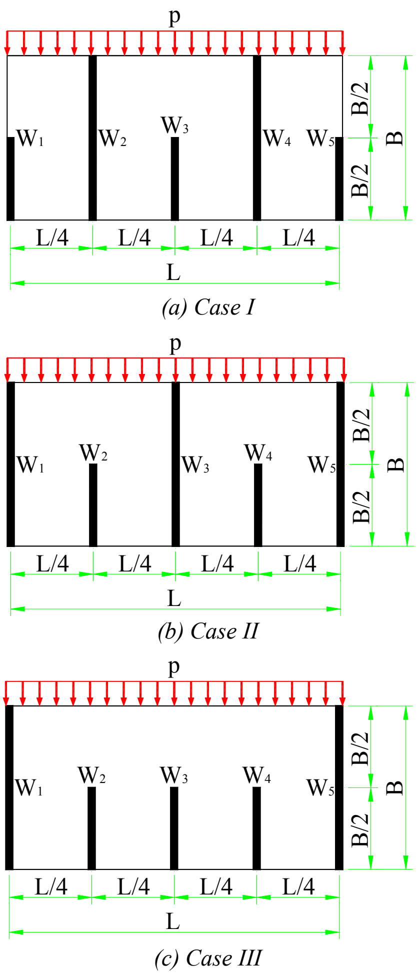


Figure 6: Three cases of building with symmetrical LLRE arrangements

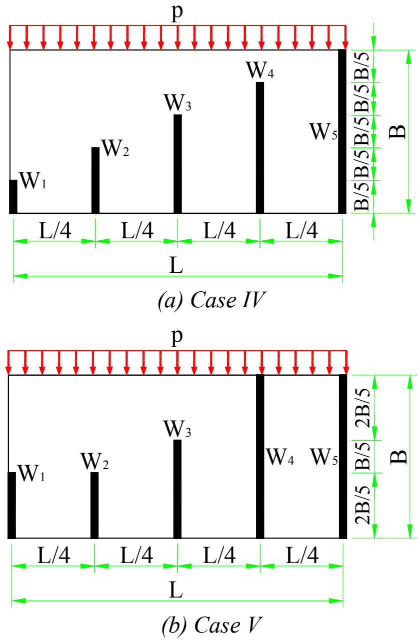


***Figure 7:** Two cases of building with asymmetrical LLRE arrangements*

Figure 6 shows three cases of LWFBs with different symmetrical LLRE arrangements under a uniform later load, *p*. In these buildings, only two types of LLRE with a length ratio of 0.5 were used. Another two cases of LWFBs with different asymmetrical LLRE arrangements are shown in Figure 7. Five types of LLRE with varying length ratios were used in these buildings. The LLRE stiffness of the LWFBs was assumed to be proportional to its length. Through increasing or decreasing the equivalent stiffness of the diaphragm, K_{DE}, n times the largest stiffness of LLREs, K_{max}, the sensitivity of load distribution to diaphragm flexibility was investigated.

3.2 REFERENCE FEA MODEL

The DBSM shown in Figure 2 was used as the reference analysis method. The DBSM analyses were performed using a FEA program. As shown in Figure 8, five FEA models, in which the deep beam was simulated using 400 two-dimensional Timoshenko beam elements, B21, and the shear wall springs was modeled using five spring elements, SPRING2, acting in the Y direction, were developed using the FEA software ABAQUS [17].

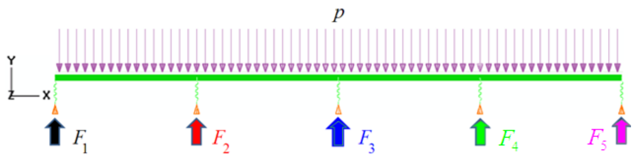
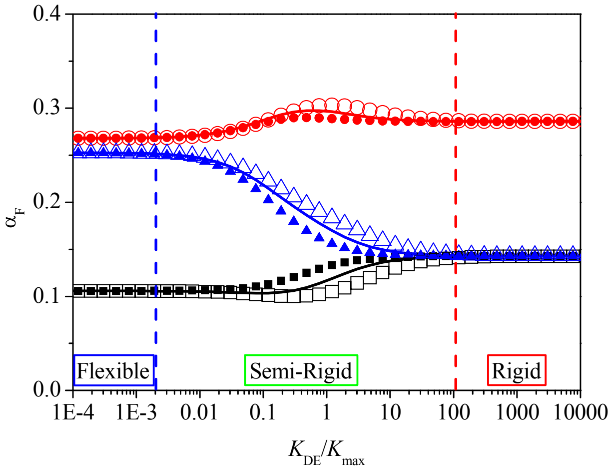


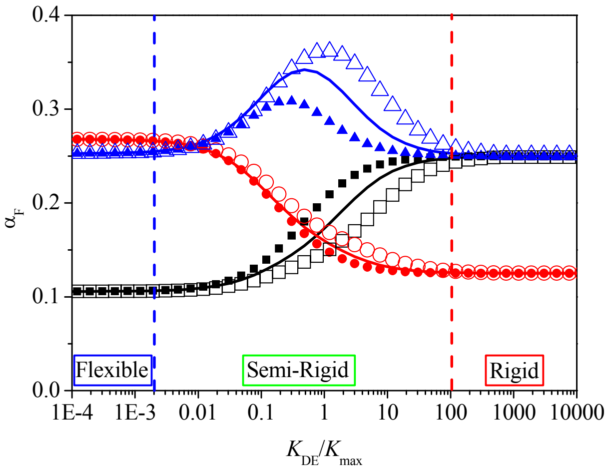
***Figure 8:** Finite element analysis model of DBSM*

3.3 SYMMETRICAL CASES

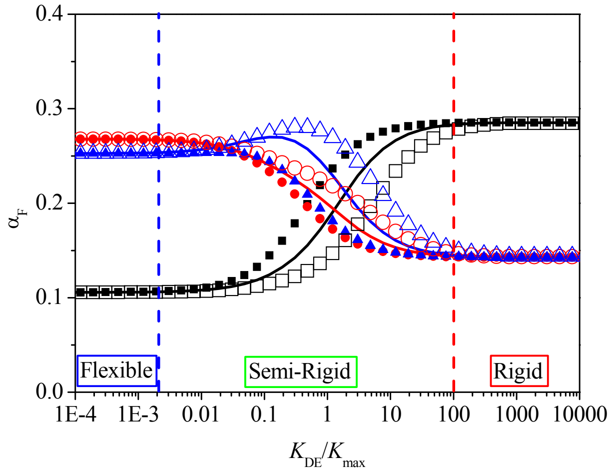
The force ratios, α_F, obtained from the DBSM were compared with ratios of reactions calculated from the MSM (Figure 3) using Equation (1) and the modified MSM (Figure 4) using Equation (8). Since the LLRE arrangements in these LWFBs is symmetrical, only F_1, F_2 and F_3 are shown in Figure 9. It shows the force ratios of the three LWFB cases predicted by the two MSMs agree well with those predicted by DBSM, especially at extreme diaphragm stiffness values. As can also be noted in Figure 9, there are more discrepancies between the two analysis methods for the semi-rigid diaphragm region. The average difference is about 0.6% with the maximum difference found being 20% for case III, regardless of using the MSM without or with rotational DOF. This level of discrepancy may be considered acceptable for structural design purposes.



(a) Case I



(b) Case II



(c) Case III

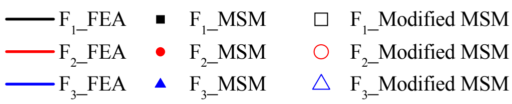


Figure 9: *Force ratio vs. stiffness ratio of symmetrical cases*

The boundaries of flexible and rigid diaphragms determined using Equations (6) and (7) are illustrated in Figure 9 as well. It shows that the range of the semi-rigid diaphragm is much wider than that assumed in design standards [6-8], and as a result, diaphragms in most buildings should be classed as semi-rigid. It is generally harder for the designers to design the LLREs with semi-rigid diaphragm than those with flexible or rigid diaphragms, since the load distribution is unknown and the solution is iterative. It is perhaps for this reason that the design standards narrow the range of the semi-rigid diaphragm with sufficient safety intentionally.

An interesting observation in the results shown in Figure 9 is that the force ratio of some of the internal springs (α_{F2} of case I and α_{F3} of cases II and III) in the semi-rigid range was found higher than the values at the two extreme regions, i.e. rigid and flexible. This phenomenon is observed in both the MSM and the FEA results. This is thought to be induced by deformation compatibility that occurs in continuous beams. Therefore, the design method using the envelope force of the two diaphragm flexibility assumptions, as proposed by APEGBC [10], may not always be conservative.

3.4 ASYMMETRICAL CASES

Figure 10 shows the comparison of the force ratios, α_F, obtained from the DBSM, the MSM without rotational DOF (Figure 3) and the modified MSM with rotational DOF (Figure 4).

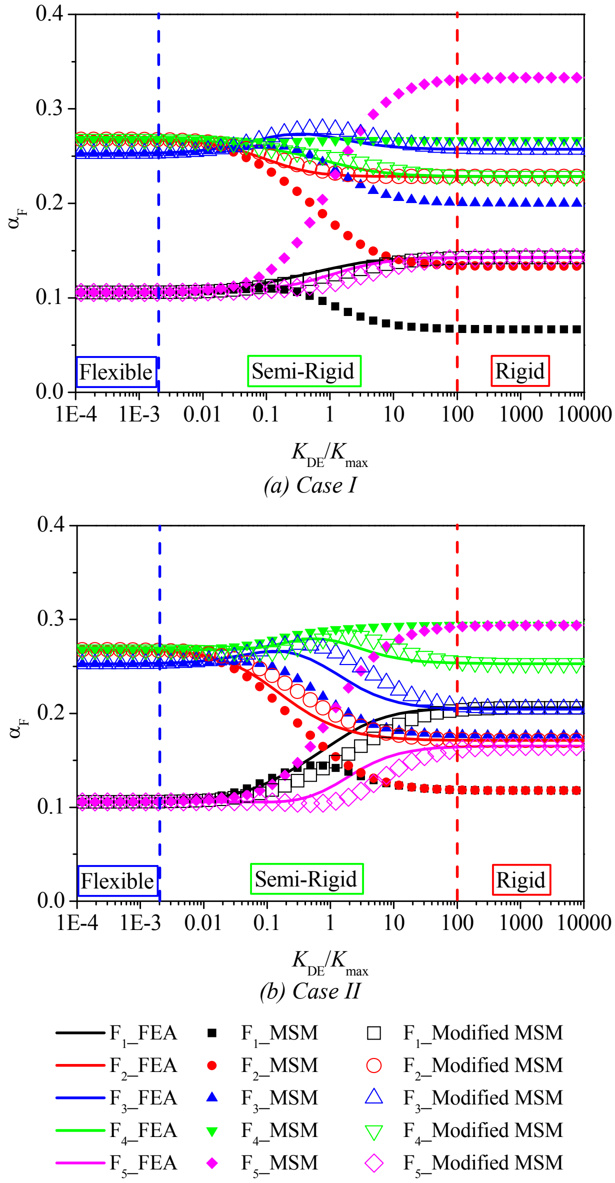


***Figure 10:** Force ratio vs. stiffness ratio of asymmetrical cases*

Figure 10 shows that the force ratios of the two LWFB cases predicted by the two MSMs agree well with those estimated by DBSM for the low diaphragm stiffness situations, where K_{DE}/K_{max} is less than 0.01. Beyond this point, the force ratios calculated by the MSM using Equation (1) deviate from those estimated by the DBSM, whereas that predicted by the modified MSM using Equation (8) still agree well with those estimated by DBSM through semi-rigid to rigid diaphragm situations.

As shown in Figure 10, the boundaries of flexible and rigid diaphragms determined using Equations (6) and (7) and illustrated in Figure 9 are still suitable for the two LWFB cases with asymmetrical arrangement of LLREs, shown in Figure 7. It seems that the arrangement of LLREs did not affect the upper and lower boundaries of semi-rigid diaphragm, even though torsional effect occurred in the asymmetrical cases. Since the K_{DE}/K_{max} of almost all the timber building was found to be in the range between 0.01 and 100, it is confirmed again that the diaphragm flexibility of timber buildings can be classified largely as semi-rigid.

The interesting observation in the results shown in Figure 8, in which the force ratio of some of the internal springs in the semi-rigid range was higher than the values at the two extreme regions, was also found in Figure 10 (α_{F3} of cases I and II and α_{F4} of case II). It indicates again that the design method using the envelope force of the two diaphragm flexibility assumptions, as proposed by APEGBC [10], may not always be conservative. Irrespective of whether the torsional effect is included or not, the modified MSM with rotational DOF proposed in this paper can be used to estimate the actual forces carried by the LLREs supporting a diaphragm, without the need to use more complex models, such as that based on DBSM.

4 CONCLUSION

In this paper, a simplified mechanics-based model (modified multiple-spring model) consisting of multiple transitional and rotational springs to represent the stiffness of diaphragms and supporting LLREs, is proposed for calculating load distribution to LLREs based on a previous MSM. Comparison between results based on the modified MSM and those obtained from previous MSM and FEA has shown that the proposed model is adequate for design use with or without accounting for the torsional effect.

The sensitive analysis provides an insight into the influence of diaphragm flexibility on the load distribution in the LLREs of timber structures. In addition, the results shown here reveal that the design method based on envelope forces may not always be conservative.

For further research, the modified multiple-spring model will be validated using test and/or further FEA results of a practical building, and the influence of the diaphragm flexibility on the torsional effect / response will be investigated.

ACKNOWLEDGEMENTS

The authors greatly acknowledge the financial support provided by Natural Sciences and Engineering Research Council (NSERC) of Canada under the Strategic Research Network on Innovative Wood Products and Building Systems (NEWBuildS).

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