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EFFICIENT MEASUREMENT OF ELASTIC CONSTANTS OF CROSS LAMINATED TIMBER USING MODAL TESTING

Jianhui Zhou[1], Ying-Hei Chui[2]

ABSTRACT: It has been shown that measurement of elastic constants of orthotropic wood-based panel products can be more efficiently measured by modal testing technique. Identification of vibration modes and corresponding natural frequencies is key to the application of modal testing technique. This process is generally tedious and requires a number of measurement locations for mode shape identification. In this study, a simplified method for frequency identification was developed which will facilitate the adoption of the vibration-based testing technique for laboratory and industrial application. In the method, the relationship between frequency order and mode order is first studied considering the boundary condition, elastic properties of the orthotropic panel. An algorithm is proposed to predict the frequency values and mode indices based on corresponding normalized sensitivity to elastic constants, initial estimates of orthotropic ratios and measured fundamental natural frequency. The output from the algorithm can be used for identification of sensitive natural frequencies from up to three frequency spectra. Then the algorithm is integrated with the elastic calculation algorithm to extract the elastic constants from the sensitive frequencies. The elastic constants of cross laminated timber panels were measured by the proposed method. The moduli of elasticity agree well with static testing results. The calculated in-plane shear modulus was found to be within the expected range.

KEYWORDS: Simultaneous frequency identification, Elastic constants, Modal testing

1 INTRODUCTION

Cross laminated timber (CLT) is made by laminating visually graded or mechanically graded structural lumber with a structural adhesive such as polyurethane, melamine and phenolic-based adhesives. CLT has been used in residential and non-residential building applications in Europe, North America and Australia since its invention in Austria and Germany in the early 1990s [1]. CLT panels are typically used as load-carrying plate elements in structural systems such as walls, floors and roofs. Key critical characteristics such as in-plane and out-of-plane bending strength, shear strength and stiffness must be taken into account in design since they affect not just safety but also serviceability performance such as deflection and vibration. Therefore, accurate measurement of its elastic properties is of great importance for predicting the mechanical behavior during structural design.

Determination of the elastic constants of both orthotropic and isotropic materials by modal testing has been proven to be a useful non-destructive testing method [2] and widely studied. Several attempts have been made for determining the in-plane elastic constants, namely E_x, E_y and G_{xy}, of solid wood plate, oriented strand board (OSB), medium density fibreboard (MDF), plywood and CLT [3-9]. The difficulties of replicating the theoretical boundary conditions in practice and identification of natural frequencies from frequency spectrum are the two main obstacles for the practical application of this technique [8]. Different boundary conditions including completely free (FFFF), one side simply supported and the other sides free (SFFF) and one side clamped and the other sides free (CFFF) were used for vibration testing. Both finite element and Rayleigh-Ritz methods were adapted for iteration or calculation to obtain experimental elastic constants based on modal testing data.

For specific boundary conditions, different elastic constants are sensitive to different vibration modes, which require the identification of the most sensitive modes for calculation. Sensitivity analysis plays an important role in the accuracy of the determination method, which should be integrated to the algorithm for extraction of elastic constants [2]. A sensitivity matrix incorporating an integration procedure was introduced and used in a study

[1] Jianhui Zhou, University of New Brunswick, 28 Dineen Drive, Fredericton, Canada. Email: jh.zhou@unb.ca
[2] Ying-Hei Chui, University of New Brunswick, 28 Dineen Drive, Fredericton, Canada. Email: yhc@unb.ca

to improve the accuracy of final results [10]. A special method of fuzzy sensitivity analysis by Hanss et al. was proposed which allowed for the determination of the inter-dependency between the material properties and the natural frequency of each mode [11]. A common method to conduct sensitivity analysis is to change 10% of each elastic constant in a finite element model (FEM) and check the difference in frequencies [7, 12]. Antunes et al. [12] found that sensitivity of natural frequencies to geometry and elastic properties was relatively low and varied quite considerably with geometry by FEM.

However, the mode order for sensitive frequencies changes between panel types. The common method for identifying vibration modes requires impact testing and tedious mode shape plotting. In order to make this technique more efficient for practical applications, this paper is aimed to identify the natural frequencies directly from the frequency spectrum. A theoretical analysis of frequency order and mode order considering the elastic constants and geometric parameters will be conducted for the determination of elastic constants of CLT panel.

2 THEORETICAL ANALYSIS

The governing differential equation for the transverse vibration of a rectangular orthotropic plate neglecting the effects of shear deformation and rotatory inertia is as follows,

$$D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2(D_1 + 2D_{xy}) \frac{\partial^4 w}{\partial x^2 y^2} + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

where D's are the flexural and torsional rigidities:

$$D_x = \frac{E_x h^3}{12(1 - v_{xy} v_{yx})},\ D_y = \frac{E_y h^3}{12(1 - v_{xy} v_{yx})},$$

$$D_1 = D_x v_{yx} = D_y v_{xy},\ D_{xy} = \frac{G_{xy} h^3}{12}.$$

E_x, E_y and G_{xy} are the in-plane elastic modulus and shear modulus, v_{xy} and v_{yx} are the Poisson's ratios, and a, b, h are the length, width and thickness of the plate, respectively, ρ is the mass density of the plate.

In this study, the boundary condition of a pair of opposite sides simply supported and the other pair free (SFSF) is chosen according to primary selections. Even though it was reported that there was analytical solution for this boundary condition [13], it is difficult to solve the transcendental frequency characteristic equations. Therefore, a closed-form approximate frequency expression was adopted from [14]. The frequency equation can be expressed as

$$f_{mn} = \frac{ab}{\pi^2} \sqrt{\frac{\rho h}{H}} \sqrt{\frac{C_{ij} + c^2 C_{\dot{m}i} + d^2 C_{\dot{m}j} - 2cE_{ij} - 2dE_{ji} + 2cdF}{1 + c^2 + d^2}} \qquad (2)$$

where f_{mn} is the frequency of mode (m, n), m and n are the number of node lines including the simply supported sides in y and x direction, respectively. It should be noted that (m, n) here is equivalent to (i, j) in [14], $\dot{m}$ and $\dot{n}$ are the same with m and n in [14], respectively. The other coefficients are expressed as follows, which can be determined using the table provided in [14]

$$H = D_1 + 2D_{xy}$$

$$C_{ij} = (D_x / H) G_x^4 r^2 + (D_y / H) G_y^4 (1/r)^2 + 2(H_x H_y + 2(D_{xy} / H)(J_x J_y - H_x H_y))$$

$$E_{ij} = H_x (K_y + L_y)(2(Dx_y / H) - 1) + 4(D_{xy} / H) J_x M_y$$

$$E_{ji} = H_y (K_x + L_x)(2(D_{xy} / H) - 1) + 4(D_{xy} / H) J_y M_x$$

$$F = -(K_x K_y + L_x L_y)(2(D_{xy} / H) - 1) + 4(D_{xy} / H) M_x M_y$$

$$c = (C_{\dot{m}j} E_{ij} - E_{ji} F) / (C_{i\dot{n}} C_{\dot{m}j} - F^2)$$

$$d = (C_{i\dot{n}} E_{ji} - E_{ij} F) / (C_{i\dot{n}} C_{\dot{m}j} - F^2)$$

The relationship between frequencies, elastic properties and geometric parameters can be summarized as:

$$f_{mn} \propto (m, n, k, h, \rho, \sigma_1, \sigma_2) \qquad (3)$$

where k is the aspect ratio $k=a/b$; σ_1 and σ_2 are the orthotropic ratios, $\sigma_1=E_y/E_x$ and $\sigma_2=G_{xy}/E_x$, respectively.

In general, h, ρ and k are constant for a given panel, σ_1 and σ_2 values are within a certain range for a specific product. For a CLT panel, σ_1 can be calculated using equation (4) and is in the range of 0.03 - 0.5 depending on lay-up configuration, which can be determined by a modified analogy to plywood method [15], σ_2 is usually about 1/16 [16].

$$\sigma_1 = \frac{E_y}{E_x} = \frac{\frac{E_{90}}{E_0} + (1 - \frac{E_{90}}{E_0}) \times \frac{h_1^3}{h_3^3}}{1 - (1 - \frac{E_{90}}{E_0}) \times \frac{h_1^3}{h_3^3}} \qquad (4)$$

where E_0 and E_{90} are the moduli of elasticity parallel and perpendicular to grain of the lamina, respectively; h_1 and h_3 are thickness of core layer and total thickness of CLT panel, respectively.

Therefore, with the input of geometric parameters and density, it is possible to propose the relationship between frequency order and mode order, namely the value f_{mn} and mode indices (m, n).

The sensitivity is defined as the proportional change in the frequency for a given proportional change in the elastic constant [2]. The normalized sensitivity of all four elastic constants to natural frequency at mode (m, n) can be expressed by

$$S_{f_{mn}/Xs} = \begin{bmatrix} \dfrac{\partial f_{mn}}{\partial E_x} \cdot \dfrac{E_x}{f_{mn}} \\ \dfrac{\partial f_{mn}}{\partial E_y} \cdot \dfrac{E_y}{f_{mn}} \\ \dfrac{\partial f_{mn}}{\partial G_{xy}} \cdot \dfrac{G_{xy}}{f_{mn}} \\ \dfrac{\partial f_{mn}}{\partial v_{xy}} \cdot \dfrac{v_{xy}}{f_{mn}} \end{bmatrix} \quad (5)$$

The normalized sensitivity is introduced and defined as the ratio of elastic modulus to E_x, and E_y and G_{xy} can be expressed by the ratio of $\sigma_1=E_y/E_x$ and $\sigma_2=G_{xy}/E_x$, respectively. The normalized sensitivity matrix can be evaluated at E_x, $E_y=\sigma_1 E_x$, $G_{xy}=\sigma_2 E_x$ and $v_{xy}=0.35$.

The initial E_x value can be calculated with the fundamental natural frequency which is always a bending mode sensitive to E_x by the approximate fundamental frequency equation [17],

$$E_{x0} = \frac{48(1-v_{xy}v_{yx})a^2 f_{20}^2 \rho}{\pi^2 h^2} \quad (6)$$

where E_{x0} is the calculated initial value of E_x, f_{20} is the fundamental frequency, a is the length of the panel, h is the thickness of the panel and ρ is panel's mass density. $(1-v_{xy}v_{yx})$ equals to 0.99 for most wood materials.

A frequency identification algorithm with normalized sensitivity analysis was developed based on the above theoretical analysis. The approximate frequency values and mode order can help identify the sensitive frequencies with the assistant of the imaginary parts of a few special impact positions. Then the frequency identification algorithm is integrated with an elastic constant calculation algorithm. The simultaneous measurement of E_x, E_y, G_{xy} and v_{xy} is achieved by minimizing the difference between measured and calculated frequencies to less than 0.1 Hz individually and a total relative frequency difference of 2.0 % by an iteration procedure as shown in Fig. 1. The algorithm is coded in MATLAB.

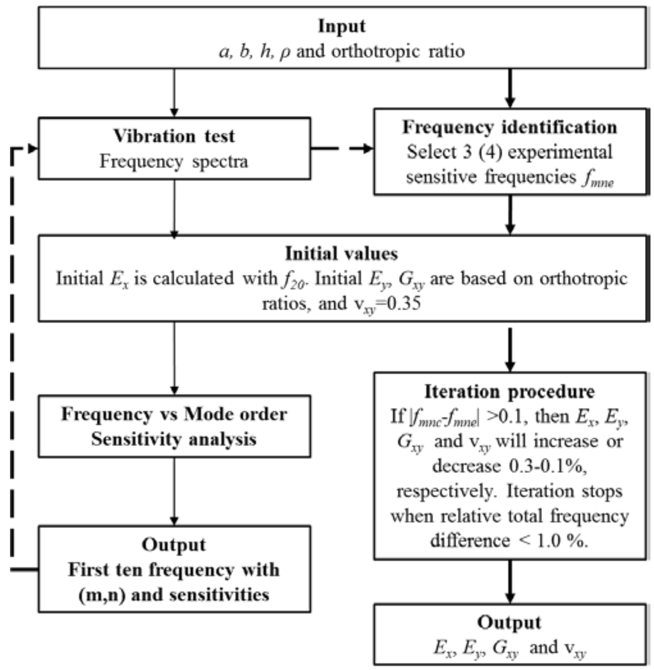

Figure 1: *Flow chart for algorithm for frequency identification and elastic constant calculation*

3 MATERIAL AND METHOD

Two full-size Canadian E1 grade three-ply SPF CLT panels of size 5.50 m × 2.15 m × 0.103 m were tested. Both panels had an average moisture content of about 12% and density of 520 kg/m^3. Due to the heavy weight of CLT panel, the CLT panel was simply resting on the steel pipes and CLT wood supports without clamping as shown in Figure 2. Two different supporting materials were chosen to study the effect of simply support condition on the frequency spectrum. The impact vibration tests were conducted on the specimen with a pair of two opposite edges parallel to minor strength direction simply supported and the other pair parallel to major strength direction free (SFSF). The accelerometer was attached at 5/12 length of one free edge, which is not on any node lines of the first ten natural frequencies. The impact and acceleration time signals were recorded by a data acquisition device and the frequency response function (FRF) was calculated from the time signals using a data analysis software. The frequency spectrum was post-processed by MATLAB software for frequency identification and calculation of the elastic constants.

The E values of CLT were measured by testing strips cut from the panel according to ASTM D4761 [18]. The cutting scheme is shown in Figure 3. Two CLT strips of size 210 mm (length) ×103 mm (height) × 89 mm (width) in each direction were cut from each CLT panel for third-point bending test as shown in Figure 4. The static G_{xy} value of CLT panel was not measured in this study.

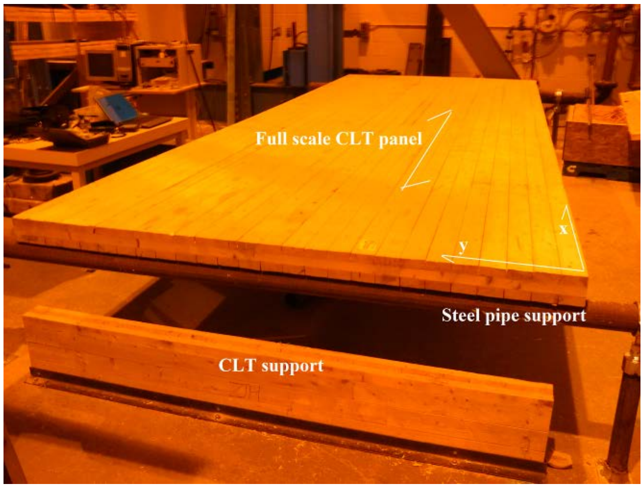

Figure 2: *Test set-up and supporting details*

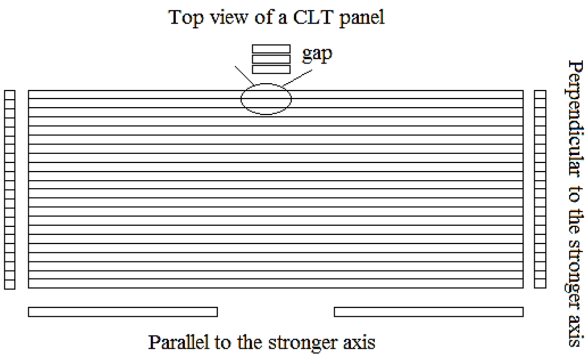

Figure 3： *Cutting scheme of CLT strips for static tests*

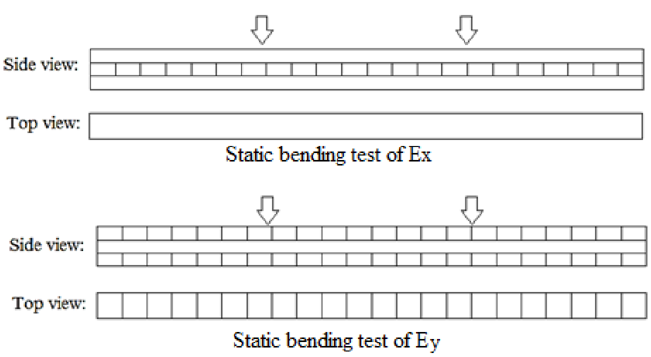

Figure 4： *Schematic of third-point bending test of CLT strips in two strength directions*

4 RESULTS AND DISCUSSION

The frequency and mode order with sensitivity analysis was conducted for the sample CLT panel. According to [16], E_{90}/E_0 and G/E_0 were assumed to be 1/30 and 1/16, respectively. The orthotropic ratios, σ_1 and σ_2, were calculated to be 0.072 and 0.0625 for the initial calculation of natural frequencies with corresponding mode indices and normalized sensitivity. The first ten natural frequencies with corresponding mode indices and normalized sensitivity to each elastic constant are listed in Table 1.

Table 1: *Frequencies with corresponding mode indices and normalized sensitivity of the first ten modes*

Frequency (Hz)	Mode indices (m , n)	Normalized sensitivity			
		E_x	E_y	G_{xy}	v_{xy}
6.73	(2, 0)	**0.498**	0.002	0.001	0.005
11.59	(2, 1)	0.168	0.007	**0.325**	0.006
26.96	(3, 0)	**0.497**	0.003	0.001	0.007
32.83	(3, 1)	0.335	0.010	0.155	0.010
33.98	(2, 2)	0.017	**0.325**	0.158	0.015
54.54	(3, 2)	0.120	0.143	0.237	0.019
60.69	(4, 0)	**0.496**	0.004	0.000	0.008
66.80	(4, 1)	0.410	0.008	0.082	0.011
80.09	(2, 3)	0.000	**0.437**	0.063	0.013
88.22	(4, 2)	0.234	0.068	0.197	0.019

Note: σ_1=0.072, σ_2=0.0625, v_{xy} =0.35, f_{20}=6.75 Hz.

It can be seen from Table 1 that the sensitive modes to E_x are modes (m, 0), and the normalized sensitivity of such modes to E_x decreases with increase in m. The most sensitive one to E_x is mode (2, 0). The sensitive modes to G_{xy} are modes (m, 1) and (m, 2), and the most sensitive one is (2, 1). The sensitive modes to E_y are modes (2, n), and the normalized sensitivity of such modes to E_y increases with increase in n. The lowest mode with relatively high sensitivity to E_y is mode (2, 2). It can also be found that the normalized sensitivity to v_{xy} of all the modes are all very small and less than 0.1, which is not sensitive enough for calculating v_{xy}. Therefore, the sensitive modes selected for determination of elastic constants are modes (2, 0), (2, 1) and (2, 2), which are the first, second and fifth in the initial calculation. The ideal mode shapes of these modes from finite element modelling are illustrated in Figure 5. It can be seen that these modes are bending mode in major strength direction, in-plane torsional mode and bending mode in minor strength direction, respectively.

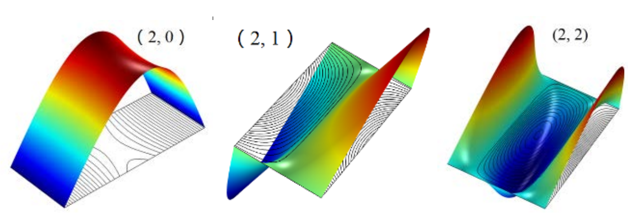

Figure 5 *Idealized mode shapes of selected sensitive modes*

The specific elastic constants are unknown before the determination, so the initial orthotropic ratios play an important role in identifying the frequency orders and mode indices. It was found under SFSF boundary condition that the first and second modes for the sample CLT panels were always modes (2, 0) and (2, 1) within a wide range of orthotropic ratios as shown in Table 2. The frequency identification process results in identifying one frequency, namely mode (2, 2) for most cases, which is the biggest advantage of SFSF compared with other boundary conditions. The frequency number of mode (2, 2) ranges from the third to the seventh within the range of orthotropic ratios considered in Table 2. Also, it can be seen that the normalized sensitivity of each elastic constant increases with any increase in its orthotropic ratio.

Table 2: *Frequency orders of sensitive modes to E_x, E_y and G_{xy} with different orthotropic ratios*

Orthotropic ratios	Sensitive modes	Frequency number	Normalized sensitivity*
σ_1=0.5 σ_2=0.1	(2, 0)	1	0.487
	(2, 1)	2	0.376
	(2, 2)	7	0.477
σ_1=0.5 σ_2=0.03	(2, 0)	1	0.488
	(2, 1)	2	0.241
	(2, 2)	7	0.512
σ_1=0.04 σ_2=0.03	(2, 0)	1	0.499
	(2, 1)	2	0.239
	(2, 2)	3	0.327
σ_1=0.04 σ_2=0.1	(2, 0)	1	0.498
	(2, 1)	2	0.363
	(2, 2)	4	0.205
	(2, 3)	8	0.352

Note: * means the sensitivity to corresponding elastic constant.

For the sample CLT panels in this study, the frequency order of mode (2, 2) will be the third to the fifth depending on the orthotropic ratio, σ_1. With the first frequency and estimated orthotropic ratios, the approximate frequency of mode (2, 2) and its mode number can be determined. By observing the positive and negative patterns of the imaginary part of the frequency response functions from up to three impact positions, the mode (2, 2) can be identified with the initial guess of the frequency value and mode order. The imaginary part patterns of the sample CLT panel can be referred to Table 3. Figure 6 shows the measurement locations.

Table 3: *Imaginary part patterns of test positions for the first seven natural frequencies*

Mode	P1	P2	P3
(2, 0)	-	-	-
(2, 1)	+	-	0
(3, 0)	-	+	0
(3, 1)	+	+	0
(2, 2)	-	-	+
(3, 2)	+	-	0
(4, 0)	-	-	+

Note: + or – represents the positive or negative of the imaginary part of the frequency response function measured from a position at a certain mode, 0 means nearly zero magnitude.

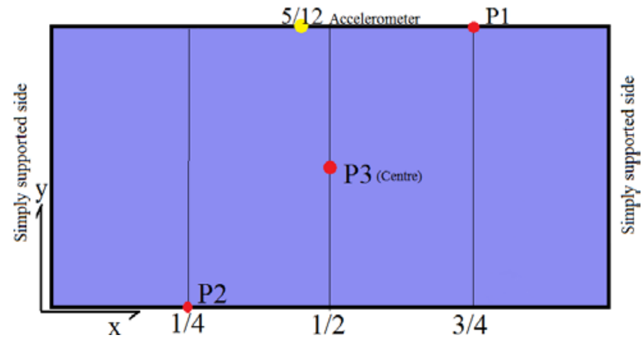

Figure 6*: Impact positions of sample CLT panel*

The sensitive natural frequencies identified from the experiments for the two panels simply supported by steel pipes and CLT supports are listed in Table 4. It can be seen that the natural frequencies of the same panel are slightly different under different supports. The frequencies of modes (2, 0) and (2, 2) under wood supports were higher than those under steel pipes supports. The reason is that the steel pipe support is non-rigid compared with rigid or semi-rigid CLT wood support. The steel pipes would bend slightly under the self-weight of CLT panel. Another minor influence is caused by the contact between CLT panel and supports. When resting on wood, the contact between panel and supports was an area with a width of 5 cm. The effective length of the sample CLT panels was about 10 cm less than the total length of CLT panel. When resting on the steel pipes, the contact between the sample CLT panel and pipes was a line. The effective length was only 3-5 cm less than the total length. However, the frequency of mode (2, 1) under wood supports was less than that under steel pipes. The natural frequencies identified would be input into the elastic constant calculation algorithm to determine the elastic constants of sample CLT panels. The frequency differences between two support conditions would lead to different calculated elastic constants. The frequency order of mode (2, 2) was identified to the third in the experiment rather than the fifth in the initial prediction. It can be explained by the over-estimation of the orthotropic ratio σ_1.

Table 4: *Identified sensitive natural frequencies of sample CLT panels*

CLT panel	Support type	Frequency (Hz)		
		(2, 0)	(2, 1)	(2, 2)
1	Steel pipe	6.75	10.13	25.38
	Wood	7.66	9.84	27.81
2	Steel pipe	6.78	10.16	24.99
	Wood	7.78	9.95	28.21

The elastic constants of CLT panel determined by modal and static tests are given in Table 5. The *E* values obtained by both methods were close to each other with an average difference of 7.5 % and 5.7 % for E_x and E_y, respectively. The differences between modal and static test results are acceptable as it was frequently reported to be around 10% [3, 7]. The modal test E_x and E_y values under steel pipes were lower than the static test values, while the modal test E_x and E_y values under wood supports were higher than the static test values. The difference between two support conditions was caused by the frequency difference discussed above. The sensitive natural frequencies to E_x and E_y, modes (2, 0) and (2, 2), under steel pipe supports were lower than that under wood supports, which caused the differences in the calculated E_x and E_y values. The E_y values obtained by modal and static bending tests were substantially lower than E_x values, which was caused by the structure of the CLT strip in the minor strength direction. The CLT panel tested in this study had no edge gluing between the laminates in both length and width directions. Gaps of 1-3 mm between laminates were observed in each layer. When the CLT strip parallel to the minor strength axis was under bending action, only the center layer can resist the force. The odd layers of CLT strip perpendicular to stronger axis essentially consist of unconnected wood blocks. Obviously there was no tensile resistance in the bottom layer. Meanwhile, there was no compressive resistance in the top layer at the initial state of bending until the curvature is small enough that adjacent laminates started to be in contact. However, the bending modulus of elasticity was calculated assuming the whole cross section of the beam can resist and transfer the force continuously. The real orthotropic ratio σ_1 was smaller than the predicted one due to the non-edge glue for the sample CLT panels.

The in-plane shear modulus was not verified in this study. There is no reference G_{xy} value of Canadian SPF CLT products. But the value measured by modal testing was thought to be reasonable compared with the values reported elsewhere. Gülzow [9, 15] reported G_{xy} value between 500 and 800 MPa for edge-glued 3-ply CLT panels made of C24/ C20 European Norway spruce with different lay-ups. With no edge glue and presence of gap between laminates, the measured in-plane shear modulus should be lower than that measured by Gülzow et al. [9, 15] for edge-glued CLT. The G_{xy} value under steel pipe supports was larger than that under wood supports due to the smaller frequency value of mode (2, 1).

Table 5: *Measured elastic constants of CLT panels by modal and static tests*

CLT panel	Test method		E_x (MPa)	E_y (MPa)	G_{xy} (MPa)
1	Modal	Steel pipe	9661 (-11.3%)	481 (-6.8%)	338
		Wood	11606 (6.6%)	508 (-1.6%)	268
	Static		10887	516	/
2	Modal	Steel pipe	9752 (-6.0%)	490 (1.2%)	341
		Wood	11006 (6.1%)	549 (13.4%)	274
	Static		10372	484	/

Note: The percent values in parentheses were using the corresponding static test value as a reference.

5 CONCLUSIONS

The proposed elastic constants determination method by modal testing under SFSF is effective to measure the elastic constants of CLT panels. The use of the SFSF boundary condition renders the process of identifying sensitive natural frequency simple. With the initial prediction of natural frequency with mode order and the imaginary part pattern of the frequency response function, the sensitive natural frequencies can be efficiently identified from the frequency response functions.

The E_x and E_y values determined by the proposed method agree well with the corresponding static values. However it was found that the effects of support conditions on the accuracy of measured elastic constants require more investigation. The G_{xy} value measured by the proposed method is reasonable compared with published values, and will need to be compared with values measured by static test method in future study.

The frequency identification algorithm highly depends on the initial guess for orthotropic ratios, especially σ_1. With specific orthotropic ratios, the natural frequency, mode order and corresponding sensitivity may be either over- or under-estimated. The algorithm will be improved to deal with uncertain orthotropic ratios in the future.

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