WCTE 2014
Quebec City Canada August 10-14
World Conference on Timber Engineering

EFFICIENT MEASUREMENT OF ELASTIC CONSTANTS OF CROSS LAMINATED TIMBER USING MODAL TESTING

Jianhui Zhou[1], Ying Hei Chui[2]

ABSTRACT: It has been shown that measurement of elastic constants of orthotropic wood-based panel products can be more efficiently measured by modal testing technique. Identification of vibration modes and corresponding natural frequencies is key to the application of modal testing technique. This process is generally tedious and requires a number of points of testing and plotting mode shapes. In this study, an auto-identification method for frequencies was developed which will facilitate the adoption of the vibration-based testing technique for laboratory and industrial application. In the method the relationship between frequency order and mode order is first studied considering the boundary condition, elastic properties and geometrical parameters of the orthotropic panel. Optimized positions of excitation and accelerometer are identified for simplifying the test procedure. Frequency spectra are processed and smoothed for picking the frequencies of interest. Cross laminated timber panels were used for experimental testing and verification.

KEYWORDS: Simultaneous frequency identification, Elastic constants, Non-destructive modal testing

1 INTRODUCTION

Determination of the elastic constants of both orthotropic and isotropic materials by modal testing has been proven to be a useful non-destructive testing method [1] and widely studied. Several attempts have been made for determining the in-plane elastic constants, namely E_1, E_2 and G_{12}, of solid wood plate, oriented strand board (OSB), medium density fibreboard (MDF), plywood and cross laminated timber (CLT) [2-8]. The difficulties of replicating the theoretical boundary conditions in practice and identification of natural frequencies from frequency spectrum are the two main obstacles for the practical application of this technique [7]. Different boundary conditions including completely free (FFFF), one side simply supported and the other sides free (SFFF), and one side clamped and the other sides free (CFFF) were used for vibration testing. Both finite element and Rayleigh-Ritz method were adapted for iteration or calculation to obtain experimental elastic constants based on modal testing data. For specific boundary conditions, different elastic constants are sensitive to different vibration modes, which requires the identification of the most sensitive modes for calculation. However, the mode order of sensitive frequencies for calculation changes from panel types. The common method for identifying vibration modes requires tedious impact testing and mode shape plotting. In order to make this technique more efficient for practical applications, this paper is aimed to identify the frequencies directly from the frequency spectrum. A theoretical analysis of frequency order and mode order considering the elastic constants and geometric parameters will be conducted as the base for spectrum analysis. Also the test procedure including impact and response positions will be optimized.

2 THEORETICAL ANALYSIS

The governing differential equation for the transverse bending of a rectangular orthotropic plate neglecting the in-plane inertia forces and rotatory inertia is as follows [9],

$$D_{11}\frac{\partial^4 w}{\partial x^4}+D_{22}\frac{\partial^4 w}{\partial y^4}+2(v_{21}D_{22}+2D_{66})\frac{\partial^4 w}{\partial x^2 y^2}+\rho h\frac{\partial^2 w}{\partial t^2}=0 \qquad (1)$$

$$D_{11}=\frac{E_1 h^3}{12(1-v_{12}v_{21})},\ D_{22}=\frac{E_2 h^3}{12(1-v_{12}v_{21})},$$

$$D_{12}=D_{11}v_{21}=D_{22}v_{12},\ D_{66}=\frac{G_{12}h^3}{12}.$$

where D's are the flexural rigidities and torsional rigidity, E_1, E_2 and G_{12} are the in-plane elastic modulus and shear modulus, v_{12} and v_{21} are the Poisson's ratios, and a, b, h are the length, width and thickness of the plate, respectively, ρ is the mass density of the plate.

[1] Jianhui Zhou, University of New Brunswick, 28 Dineen Drive, Fredericton, Canada. Email: jh.zhou@unb.ca
[2] Yinghei Chui, University of New Brunswick, 28 Dineen Drive, Fredericton, Canada. Email: yhc@unb.ca

For the four sides simply supported case, there is a close form solution to Eq. 1, the frequency equation can be expressed as:

$$f_{mn} = \frac{\pi}{2a^2b^2}\left(\frac{1}{\rho h}\left[D_{11}m^4b^4 + D_{22}n^4a^4 + 2(D_{12}+2D_{66})m^2n^2a^2b^2\right]\right)^{\frac{1}{2}} \quad (2)$$

For the other kinds of boundary conditions, there are no close form solutions. The Rayleigh-Ritz method was used to obtain an approximate frequency equation:

$$f_{mn} = \frac{1}{2\pi\sqrt{\rho h}}\left[D_{11}\frac{r_1(m,n)}{a^4} + D_{22}\frac{r_2(m,n)}{b^4} + 2D_{12}\frac{r_3(m,n)}{a^2b^2} + 4D_{66}\frac{r_4(m,n)}{a^2b^2}\right]^{\frac{1}{2}} \quad (3)$$

where (m, n) is the mode indices, and a, b, h are the length, width and thickness of the plate, respectively and r's are the coefficient values for different modes.

Therefore, the relationship between frequencies, elastic properties and geometric parameters can be summarized as:

$$f_{mn} \propto (m, n, k, h, \rho, \sigma_1, \sigma_2) \quad (4)$$

where k is the aspect ratio $k = a/b$, σ_1 and σ_2 are defined as the ratios of elastic constants, $\sigma_1 = E_2/E_1$, $\sigma_2 = G_{12}/E_1$.

In general h and r are constant, and k, σ_1 and σ_2 values are within a certain range for a specific product. For example, the k value of CLT panels is in range from 4 to 6, and σ_1 is in the range from 0.5 - 1, σ_2 is around 1/16. Therefore, with the input of geometric parameters and density, it is possible to propose the relationship between frequency order and mode order, namely the f_{mn} value and (m, n) indices. The threshold values for k, σ_1 and σ_2 can be found for any change in mode order.

3 MATERIAL AND METHOD

CLT panels of different aspect ratios were used for completely free vibration tests. A fine grid was formed for impact vibration tests with a hammer and an accelerometer. The frequency spectrum can be obtained by an LSD Dactron acquisition device with a signal analysis software RT Pro 6.33.

The impact points were reduced based on the mode lines from the first mode to the highest mode of interest. The positions of impact and accelerometer were optimized in order for a clear frequency spectrum including all the modes up to the highest mode of interest.

The frequency spectrum was post-processed by MATLAB software for peak-picking and calculation of the elastic constants.

4 RESULTS AND DISSCUSSION

Test data are being collected and analysed, and the results will be presented in the final paper.

5 CONCLUSIONS

The method proposed in this paper can be used for simultaneously identifying the frequencies of interests for measuring the elastic constants of cross laminated timber panel and other panel-shaped materials. The relationship of frequency order and mode order can be implemented into a computer program for predicting mode indices with initial inputs such as density, geometric parameters and approximate orthotropic ratios.

ACKNOWLEDGEMENT

The authors greatly acknowledge the financial support provided by Natural Sciences and Engineering Research Council (NSERC) of Canada under the Strategic Research Network on Innovative Wood Products and Building Systems (NEWBuildS).

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